PRODUCTION AND OPERATIONS MANAGEMENT

2023/2024

Lisbon School of Economics & Management

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Inventory Management

Chapter 3

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Agenda

- The Importance of Inventory
- Types of Inventory
- Functions of Inventory
- EOQ Model
- POQ Model
- Quantity Discount Models
- Probabilistic Models and Safety Stock

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Inventory Management

- The objective of inventory management is to strike a balance between **inventory investment** and **customer service**

The Importance of Inventory

- Inventory is one of the most expensive assets of many companies, representing as much as 50% of total invested capital.
- On the one hand, a firm can reduce costs by reducing inventory. On the other hand, production may stop and customers become dissatisfied when an item is out of stock.
- ***The objective of inventory management is to strike a balance between inventory investment and customer service.***

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Functions of Inventory

1. To decouple or separate various parts of the production process
2. To decouple the firm from fluctuations in demand and provide a stock of goods that will provide a selection for customers
3. To take advantage of quantity discounts
4. To hedge against inflation

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Types of Inventory

- Raw material
  - Purchased but not processed
- Work-in-process (WIP)
  - Undergone some change but not completed
  - A function of cycle time for a product
- Maintenance/repair/operating (MRO)
  - Necessary to keep machinery and processes productive
- Finished goods
  - Completed product awaiting shipment

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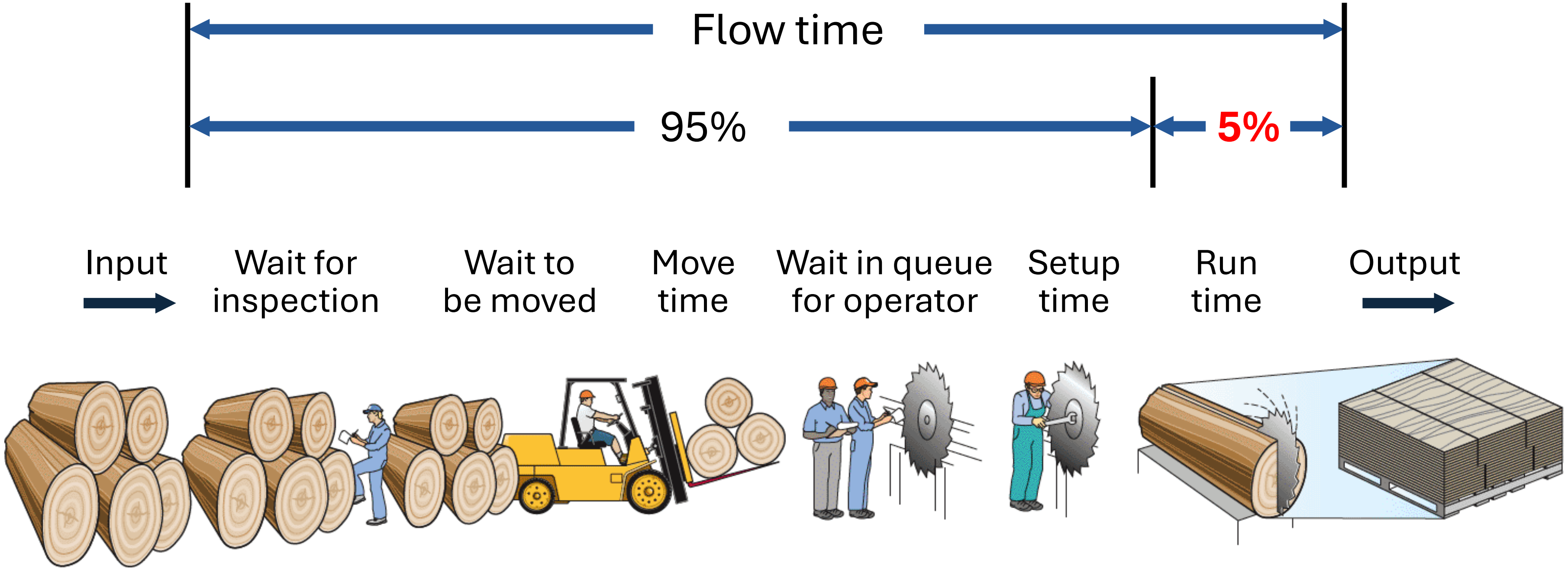
The Material Flow Cycle

Most of the time that work is in-process (**95% of the cycle time**) is not productive time.



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Why Stocks?

- Economies of scale → *Cyclical Stock*
- Demand variability
- Information variability → *Safety Stock*
- Seasonality → *Seasonal Stock*
- Quantity discounts → *Strategic Stock*

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Managing Inventory

1) How inventory items can be classified (called ABC analysis)?
2) How accurate inventory records can be maintained?

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ABC Analysis (Pareto principle)

- Divides inventory into three classes based on annual dollar volume
  - Class A - high annual dollar volume
  - Class B - medium annual dollar volume
  - Class C - low annual dollar volume
- Used to establish policies that **focus on the *few critical* parts** and not the many trivial ones.
- Greater control of class A products.

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Example: ABC Analysis

Item Stock Number	Percent of Number of Items Stocked	Annual Volume (units)	X	Unit Cost	=	Annual Dollar Volume	Percent of Annual Dollar Volume	Class
#10286	20%	1,000		$ 90.00		$ 90,000	38.8%	A
#11526		500		154.00		77,000	33.2%	A
#12760		1,550		17.00		26,350	11.3%	B
#10867	30%	350		42.86		15,001	6.4%	B
#10500		1,000		12.50		12,500	5.4%	B

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Example: ABC Analysis

Item Stock Number	Percent of Number of Items Stocked	Annual Volume (units)	x	Unit Cost	=	Annual Dollar Volume	Percent of Annual Dollar Volume	Class
#12572		600		$ 14.17		$ 8,502	3.7%	C
#14075		2,000		.60		1,200	.5%	C
#01036	50%	100		8.50		850	.4%	C
#01307		1,200		.42		504	.2%	C
#10572		250		.60		150	.1%	C
		8,550				$232,057	100.0%	

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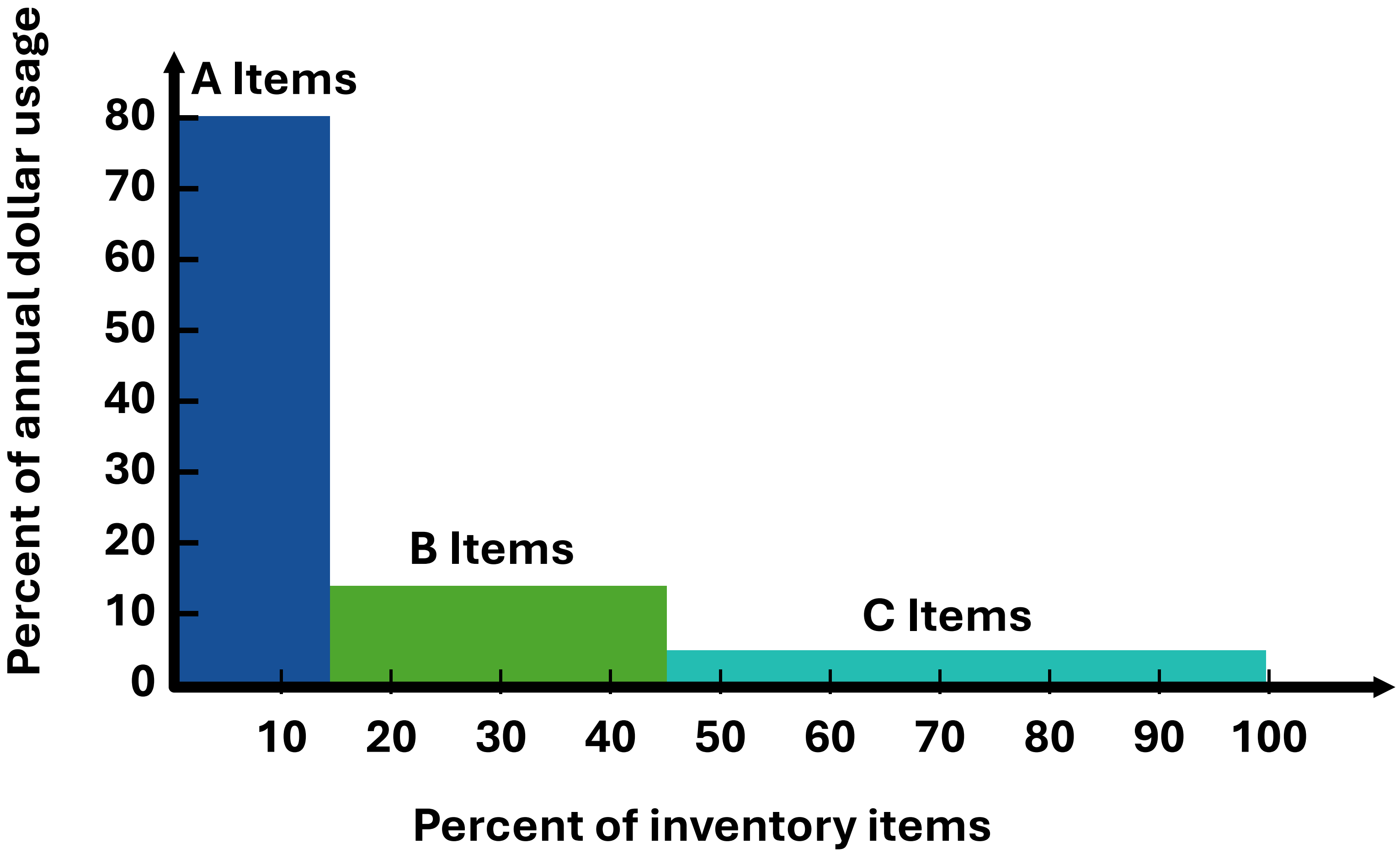
Example: ABC Analysis



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ABC Analysis

- Other criteria than annual dollar volume may be used
  - Anticipated engineering changes
  - Delivery problems
  - Quality problems
  - High unit cost (stock or stockout)

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ABC Analysis

- Policies employed may include
  - More emphasis on supplier development for A items
  - Tighter physical inventory control for A items
  - More care in forecasting A items

Better forecasting, physical control, supplier reliability, and an ultimate reduction in inventory can all result from classification systems such as ABC analysis.

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Record Accuracy

- Regardless of the inventory system, record accuracy requires good incoming and outgoing record keeping as well as good security.
- Stockrooms will have limited access, good housekeeping, and storage areas that hold fixed amounts of inventory.
- Meaningful decisions about ordering, scheduling, and shipping, are made only when the firm knows what it has on hand.

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Record Accuracy

- Record accuracy is a prerequisite to inventory management, production scheduling, and, ultimately, sales. Accuracy can be maintained by either ***periodic*** or ***perpetual*** systems.
- ***Periodic systems*** require regular (periodic) checks of inventory to determine quantity on hand.
- ***Perpetual inventory*** tracks both receipts and subtractions from inventory on a continuing basis.

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Independent versus Dependent Demand

- **Independent demand** - the demand for item is independent of the demand for any other item in inventory
- **Dependent demand** - the demand for item is dependent upon the demand for some other item in the inventory (e.g.: automobile parts)

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Holding, Ordering and Setup Costs

- **Holding costs, H** - the costs of holding or "carrying" inventory over time (one year)
- **Ordering costs, S** - the costs of placing an order and receiving goods
- **Setup costs, S** - cost to prepare a machine or process for manufacturing an order

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Periodic revision (*Cycle Counting*)

- Articles are counted and records are updated periodically Often used with A B C analysis
- It has several advantages:
    1. Eliminates stops and interruptions
    2. Eliminates annual inventory adjustment
    3. Trained personnel audit inventory accuracy
    4. Allows the causes of errors to be identified and corrected
    5. Maintains accurate inventory records

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Example of periodic revision (*Cycle Counting*)

- 5000 items in stock: 500 A items, 1750 B items, 2750 C items.
- The policy is to count A items every month (20 working days), B items every quarter (60 days) and C items every six months (120 days)

Item	Quantity	Counting policy	Number of items counted per day
A	500	Every mounth	500/20 = 25/day
B	1750	Every quarter	1,750/60 = 29/day
C	2750	All semesters	2,750/120 = 23/day
Total			77/day

Example of periodic revision (*Cycle Counting*)

Minimizing Costs (3 of 6)

Q = Number of units per order

Q^* = Optimal number of units per order (EOQ)

D = Annual demand in units for the inventory item

S = Setup or ordering cost for each order

H = Holding or carrying cost per unit per year

$$\text{Annual setup cost} = (\text{Number of orders placed per year}) \times (\text{Setup or order cost per order})$$

$$= \left(\frac{\text{Annual demand}}{\text{Number of units in each order}}\right)(\text{Setup or order cost per order})$$

$$= \left(\frac{D}{Q}\right)S$$

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Example of periodic revision (*Cycle Counting*)

Minimizing Costs (4 of 6)

Q = Number of units per order

Q^* = Optimal number of units per order (EOQ)

D = Annual demand in units for the inventory item

S = Setup or ordering cost for each order

H = Holding or carrying cost per unit per year

$$\text{Annual setup cost} = \frac{D}{Q}S$$

$$\begin{aligned}\text{Annual setup cost} &= (\text{Number of orders placed per year}) \\ &\quad \times (\text{Setup or order cost per order}) \\ &= \left(\frac{\text{Annual demand}}{\text{Number of units in each order}}\right)(\text{Setup or order cost per order}) \\ &= \left(\frac{D}{Q}\right)S\end{aligned}$$

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Example of periodic revision (*Cycle Counting*)

Minimizing Costs (5 of 6)

Q = Number of units per order

Q^* = Optimal number of units per order (EOQ)

D = Annual demand in units for the inventory item

S = Setup or ordering cost for each order

H = Holding or carrying cost per unit per year

$$\text{Annual setup cost} = \frac{D}{Q}S$$

$$\text{Annual holding cost} = \frac{Q}{2}H$$

$$\begin{aligned}\text{Annual holding cost} &= (\text{Average inventory level}) \\ &\quad \times (\text{Holding cost per unit per year}) \\ &= \left(\frac{\text{Order quantity}}{2}\right)(\text{Holding cost per unit per year}) \\ &= \left(\frac{Q}{2}\right)H\end{aligned}$$

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Example of periodic revision (*Cycle Counting*)

Minimizing Costs (6 of 6)

Q = Number of units per order

Q^* = Optimal number of units per order (EOQ)

D = Annual demand in units for the inventory item

S = Setup or ordering cost for each order

H = Holding or carrying cost per unit per year

$$\text{Annual setup cost} = \frac{D}{Q}S$$

$$\text{Annual holding cost} = \frac{Q}{2}H$$

Optimal order quantity is found when annual setup cost equals annual holding cost Solving for Q^*

$$\left(\frac{D}{Q}\right)S = \left(\frac{Q}{2}\right)H$$

$$2DS = Q^2H$$

$$Q^2 = \frac{2DS}{H}$$

$$Q^* = \sqrt{\frac{2DS}{H}}$$

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Examples: Holding Costs

- Obsolescence
- Insurances
- Staffing
- Taxes
- Pilferage
- Depreciation
- Material handling costs
- Etc.

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Examples: Holding Costs

Category	Cost (and range) as a Percent of Inventory Value
Housing costs (building rent or depreciation, operating costs, taxes, insurance)	6% (3 - 10%)
Material handling costs (equipment lease or depreciation, power, operating cost)	3% (1 - 3.5%)
Labor cost	3% (3 - 5%)
Investment costs (borrowing costs, taxes, and insurance on inventory)	11% (6 - 24%)
Pilferage, space, and obsolescence	3% (2 - 5%)
Overall carrying cost	26%

Examples: Holding Costs

Category	Cost (and range) as a Percent of Inventory Value
Housing costs (building rent or depreciati... i...	
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In... an... ...y))	... (... - 24%)
Pilferage, space, and obsolescence	3% (2 - 5%)
Overall carrying cost	26%

Holding costs vary considerably depending on the business, location, and interest rates. Generally greater than 15%, some high tech items have holding costs greater than 40%.

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Examples: Ordering and Setup Costs

Ordering costs

- Documents
- Supplies
- Order processing
- Administrative support
- Etc.

Setup costs

- Cleaning
- Re-tooling
- Adjustments
- Etc.

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Independent Demand Models

How much and **When** to order?

- **Deterministic Models**
  - *Economic Order Quantity (EOQ)*
  - *Production Order Quantity (POQ)*
  - *Quantity Discount*

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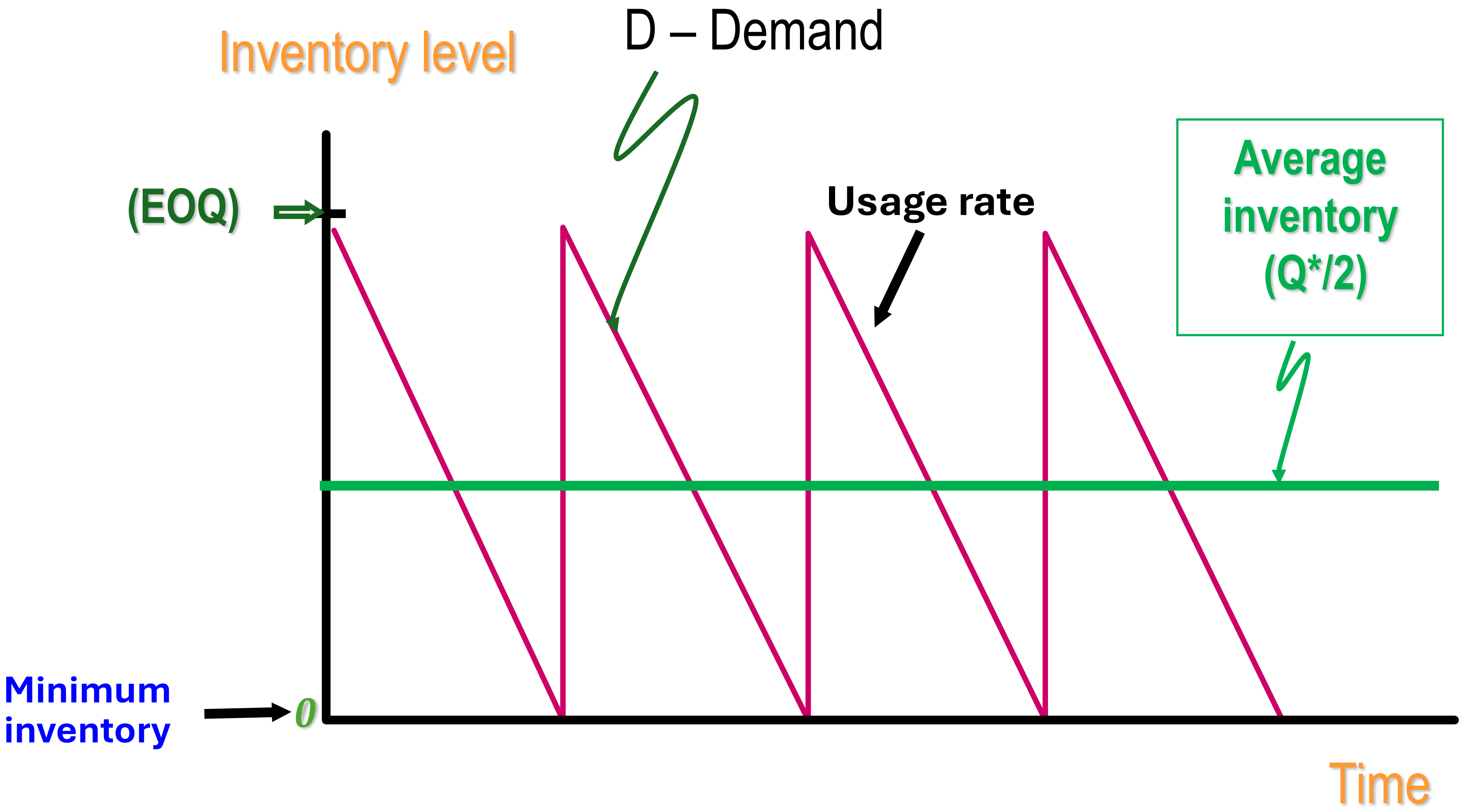
EOQ – Important assumptions

1. Demand is known, constant, and independent
2. Lead time is known and constant
3. Receipt of inventory is instantaneous and complete
4. Quantity discounts are not possible
5. Only variable costs are setup and holding
6. Stockouts can be completely avoided

EOQ model (Wilson)



Annual number of orders = **D/Q**

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EOQ model

Economic Order Quantity = $Q^* = \sqrt{\frac{2DS}{H}}$

Maximum inventory = Q

Order cost = $\frac{D}{Q} S$

Holding cost = $H \frac{Q}{2}$

D = Annual demand
S = Setup/Ordering cost
H = Holding cost/unit/year

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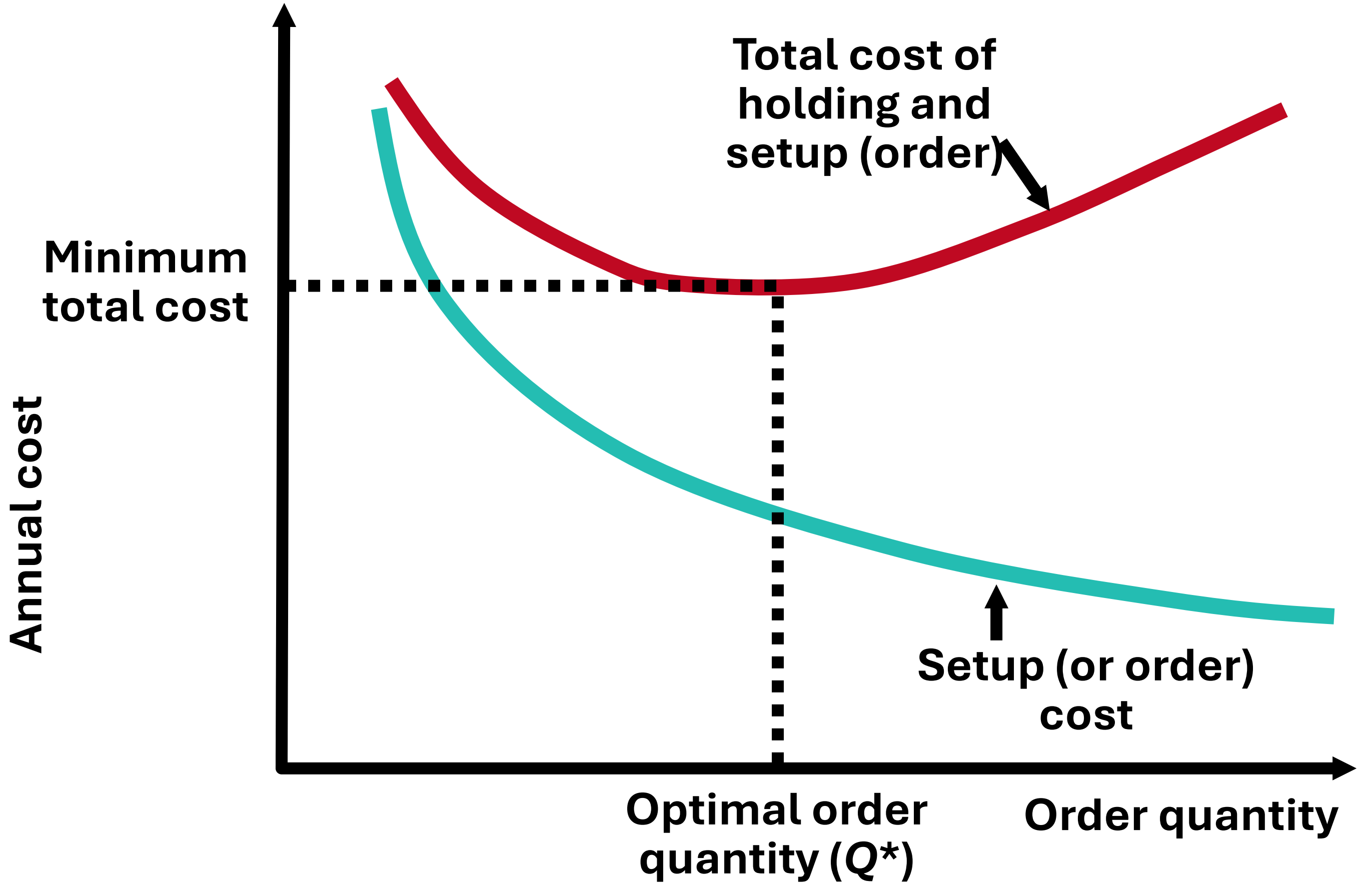
Minimizing Costs

Objective is to minimize total costs



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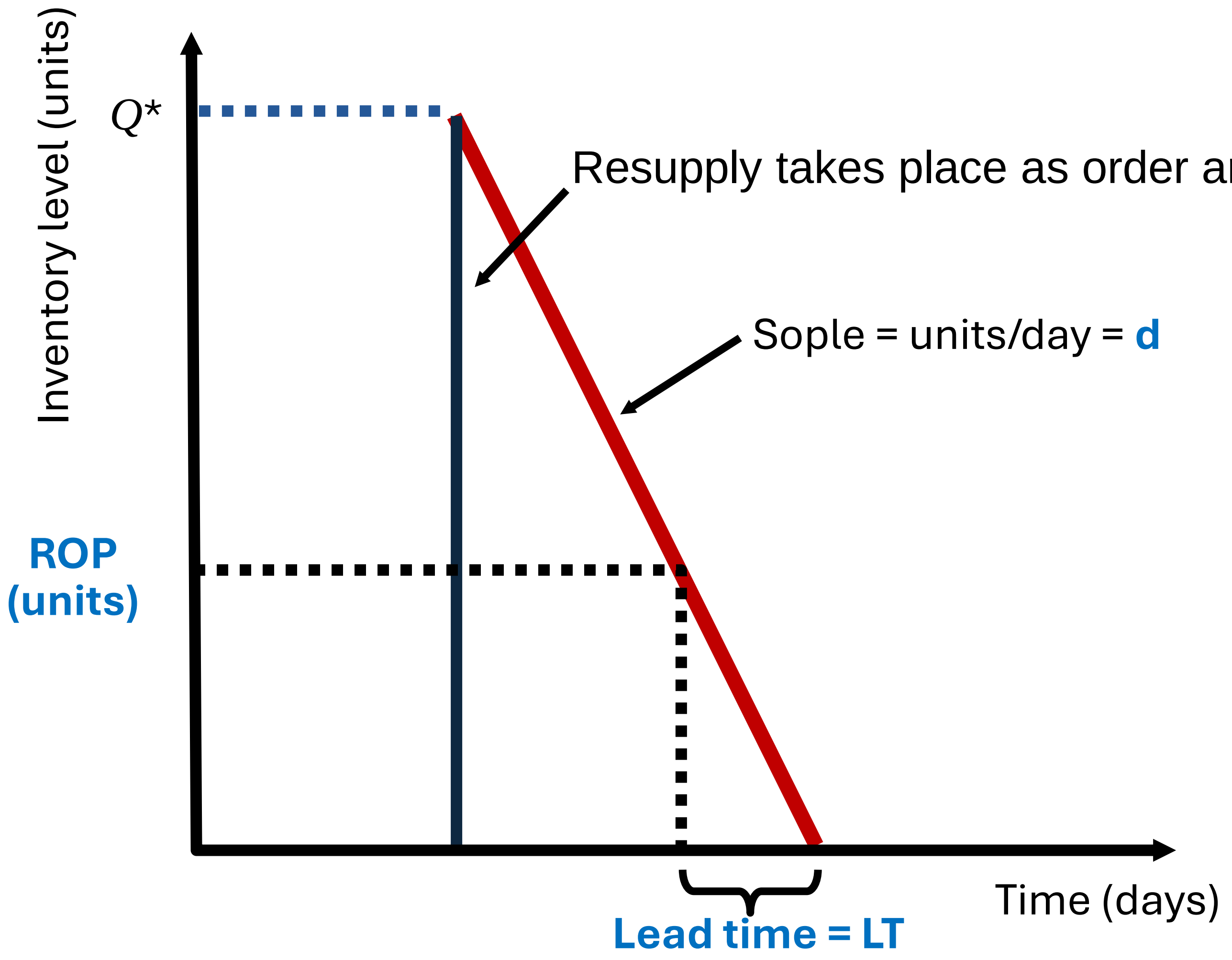
Reorder Point - ROP



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POQ model

- Used when inventory builds up over a period of time after an order is placed
- Used when units are produced and sold simultaneously
- Restocking is not instantaneous

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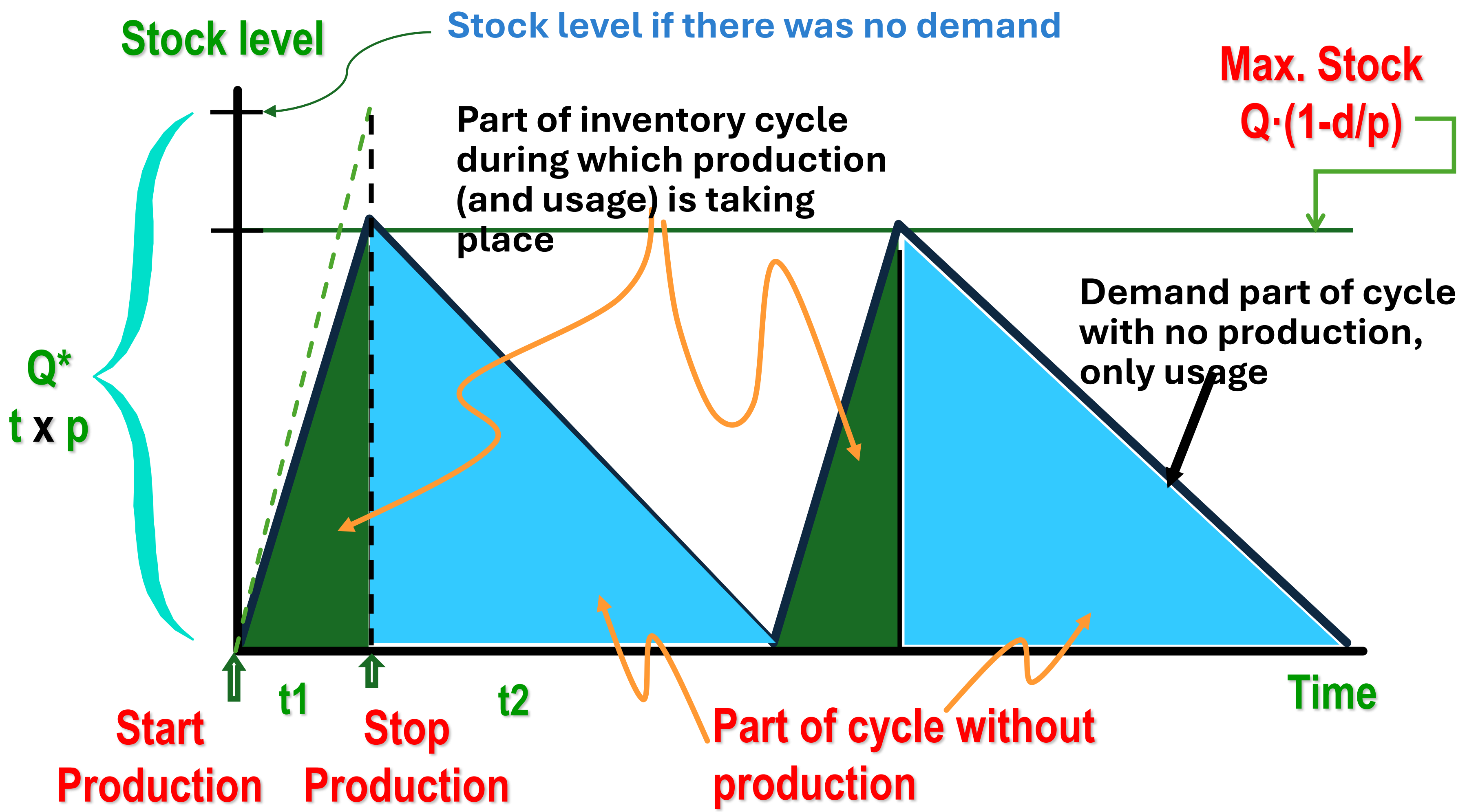
POQ model



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POQ model

Q = Number of units per order
H = Holding cost per unit per year
t = Length of the production run in days
p = Production rate (daily, weekly, monthly)
d = Demand rate (daily, weekly, monthly)

(Annual inventory holding cost) = (Average inventory level) x (Holding cost per unit per year)

(Annual inventory level) = (Maximum inventory level)/2

(Maximum inventory level) = (Total produced during the production run) - (Total used during the production run)

$$= pt - dt \text{ or } Q \times (1 - d/p)$$

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Reorder Point - ROP

- EOQ answers the question **"How much"** to order
- ROP defines **"When"** to order
- **Lead Time (LT)** defines the time between placement and receipt of an order

$$\textbf{ROP} = \left(\text{Demand per day}\right)\left(\text{Lead time (in days)}\right)$$

$$\textbf{ROP} = \textbf{d} \times \textbf{LT}$$

$$\textbf{d} = \frac{D}{\text{Number of working days in an year}}$$

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POQ model

Production order quantity $= Q_p^* = \sqrt{\dfrac{2DS}{H\left(1 - \dfrac{d}{p}\right)}}$

Maximum inventory $= Q\left(1 - \dfrac{d}{p}\right)$

Setup cost $= \dfrac{D}{Q} S$

Holding cost $= (0,5)HQ\left(1 - \dfrac{d}{p}\right)$

D = Annual demand

S = Setup cost

H = Holding cost/unit/year

d = Daily demand (weekly, monthly)

p = Daily production (weekly, monthly)

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Quantity Discount Model

- Reduced prices are often available when larger quantities are purchased
- Trade-off is between reduced product cost and increased holding cost

Total Cost = Setup cost + Holding cost + Product cost

TC = (D/Q)*S + (Q/2)*(I*P) + P*D

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Quantity Discount Model

- Note that holding cost is *IP* instead of *H* as seen in the regular EOQ model. Because the price of the item is a factor in annual holding cost, we do not assume that the holding cost is a constant when the price per unit changes for each quantity discount. Thus, it is common to express the **holding cost as a percent (*I*) of unit price (*P*)** when evaluating costs of quantity discount schedules.

$$TC = (D/Q)*S + (Q/2)*(I*P) + P*D$$

Where:

D = Annual demand

Q = Quantity ordered

S = Ordering cost

P = Price per unit

I = Holding cost per unit per year expressed as a percent of price P

The EOQ formula is modified for the quantity discount problem as follows:

$$Q^* = \sqrt{2DS/IP}$$

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Quantity Discount Model

Steps in analyzing a quantity discount

1. For each discount, calculate *Q**
2. If *Q** for a discount doesn't qualify, choose the smallest possible order size to get the discount
3. Compute the total cost for each *Q** or adjusted value from Step 2
4. Select the ***Q**** that gives the **lowest total cost**

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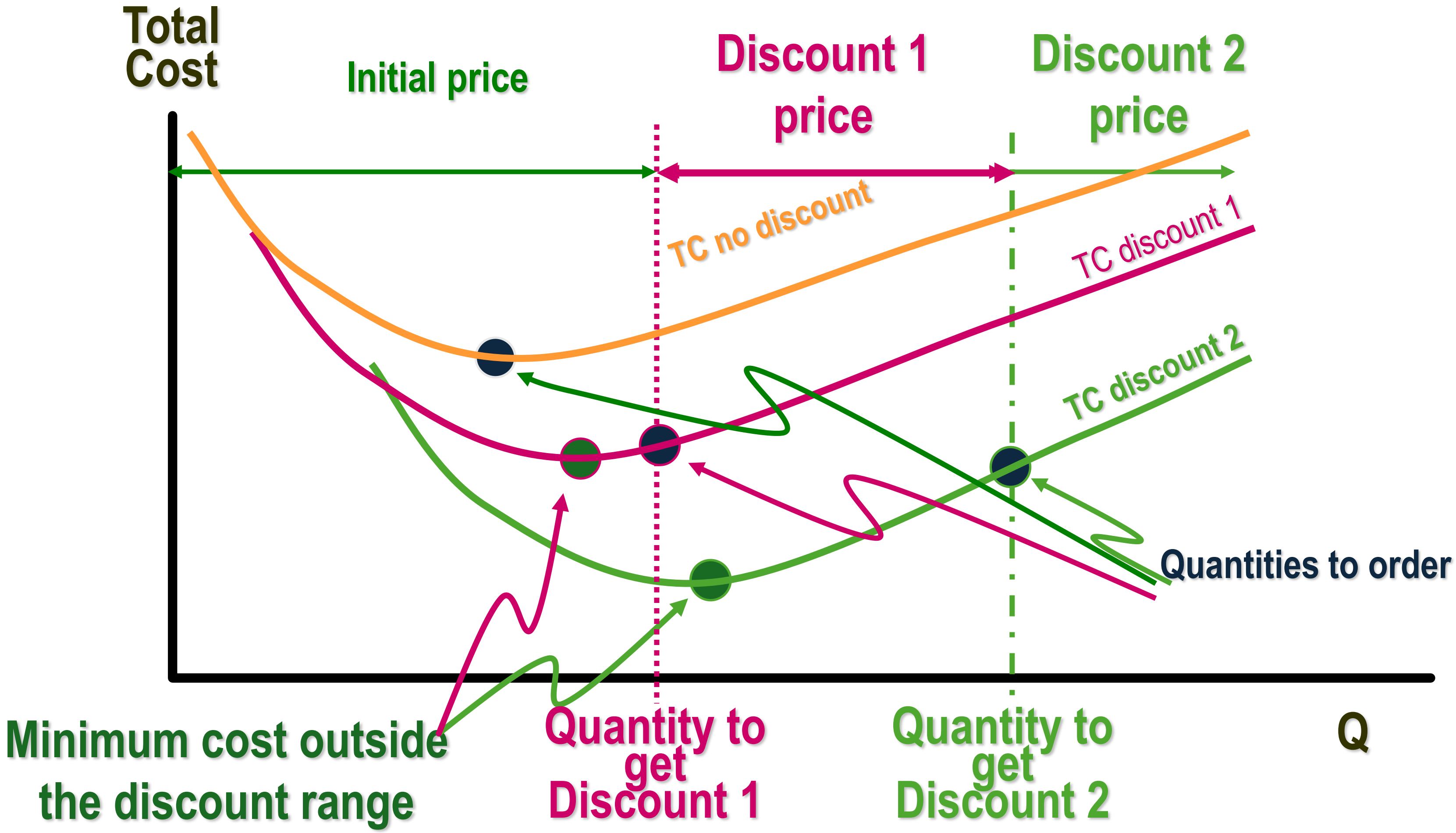
Quantity Discount Model



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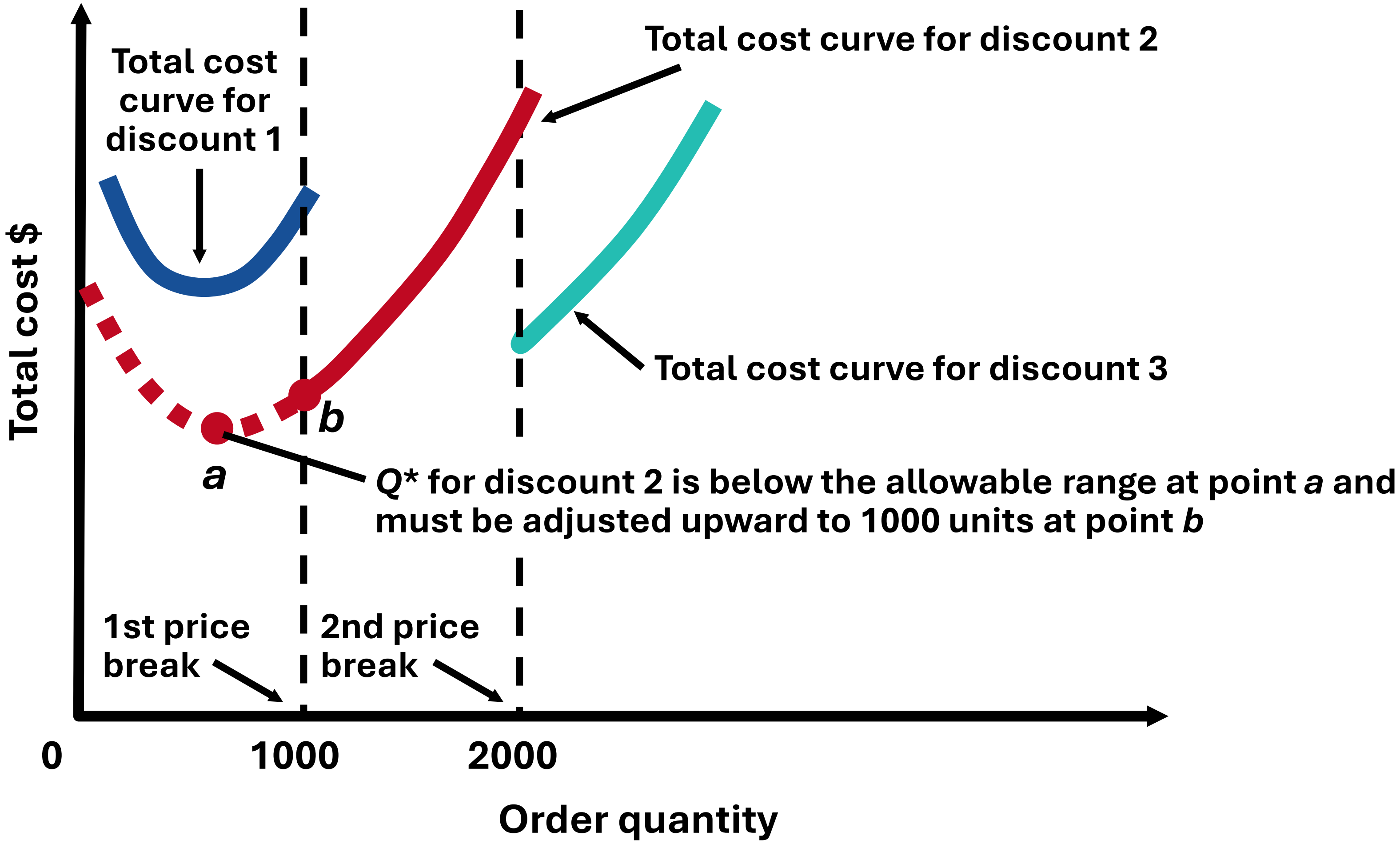
Quantity Discount Model



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Quantity Discount Model - Example

- Answers the question of when and how much to order
- Allows quantity discounts:
  - Lower price when large quantities are purchased
  - Remaining assumptions of the EOQ model
- Trade-off between lower acquisition costs and higher ownership costs

Considering that D = 5200 units, S = $200, and I = 28%, and the information in the following table, calculate the quantity to order.

	Quantity Ordered	Price per unit *P*
Initial Price	0 to 119	$ 100
Discount price 1	120 to 1499	$ 98
Discount price 2	1500 and over	$ 96

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Quantity Discount Model

	Quantity Ordered	Price per unit *P*
Initial Price	0 to 119	$ 100
Discount price 1	120 to 1499	$ 98
Discount price 2	1500 and over	$ 96

Solution Procedure:

STEP 1: Starting with the ***lowest*** possible purchase price in a quantity discount schedule and working toward the highest price, keep calculating Q^* until the first feasible EOQ is found. The first feasible EOQ is a possible best order quantity, along with all price-break quantities for all *lower* prices.

STEP 2: Calculate the total annual cost *TC* for each of the possible best order quantities determined in Step 1. **Select the quantity that has the lowest total cost.** Note that no quantities need to be considered for any prices greater than the first feasible EOQ found in Step 1. This occurs because if an EOQ for a given price is feasible, then the EOQ for any *higher* price *cannot* lead to a lower cost (*TC* is guaranteed to be higher).

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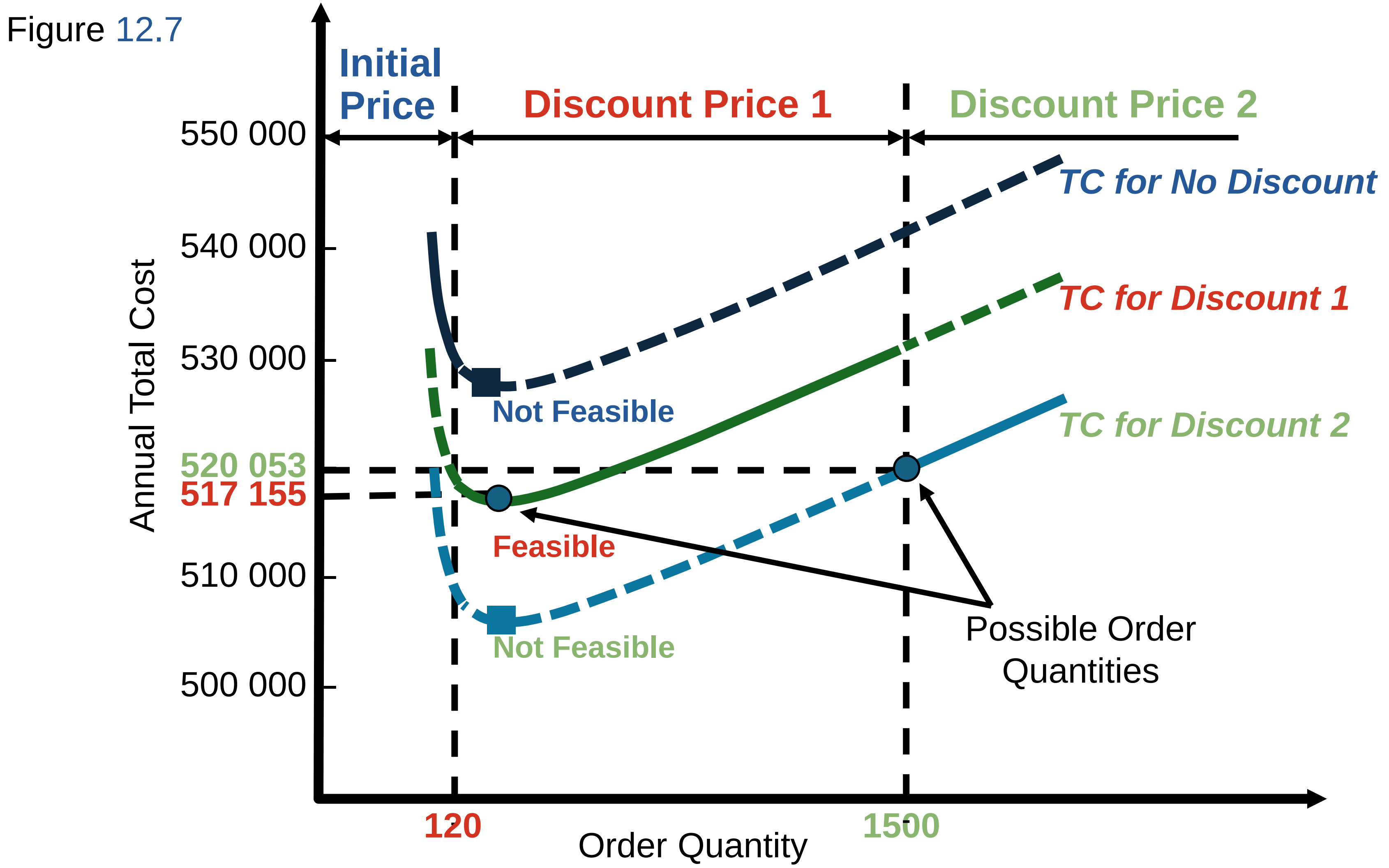
Quantity Discount Model - Example



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Quantity Discount Model - Example

First we calculate the Q^* for the lowest possible price of $96:

$Q^*_{\$96} = \sqrt{(5200)(\$200)/(0.28)(\$96)}$ = **278** flying drones per order

Because 278 < 1500, this EOQ is ***infeasible*** for the $96 price.

So now we calculate Q^* for the next-higher price of $98:

$Q^*_{\$98} = \sqrt{(5200)(\$200)/(0.28)(\$98)}$ = **275** flying drones per order

Because 275 is between 120 and 1499 units, this EOQ is ***feasible*** for the $98 price.

$$Q^* = \sqrt{\frac{2DS}{IP}}$$

Non feasible – calculate Q* for the next price

Feasible

Thus, the possible best order quantities are 275 (the first feasible EOQ), and 1500 (the price-break quantity for the lower price of $96).

We need not bother to compute Q^* for the initial price of $100 because we found a feasible EOQ for a lower price.

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Quantity Discount Model - Example

Total Cost Computations

ORDER QUANTITY	UNIT PRICE	ANNUAL ORDERING COST	ANNUAL HOLDING COST	ANNUAL PRODUCT COST	TOTAL ANNUAL COST
275	**$98**	$3782	$3773	$509 600	**$517 155**
1500	$96	$693	$20 160	$499 200	$520 053

Choose the price and quantity that gives the lowest total cost.

Order 275 drones at $98 per unit.

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Probabilistic Models and Safety Stock

- Used when demand is not constant or uncertain
- Use safety stock to achieve the desired service level and avoid stockouts

Use prescribed service levels to set safety stock when the cost of stockouts cannot be determined

$$ROP = demand\ during\ Lead\ Time + Z\sigma_{dLT}$$

Where Z = number of standard deviations

σ_{dLT} = Standard Deviation during Lead Time

($Z\sigma_{dLT}$ = Safety Stock)

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Probabilistic Demand

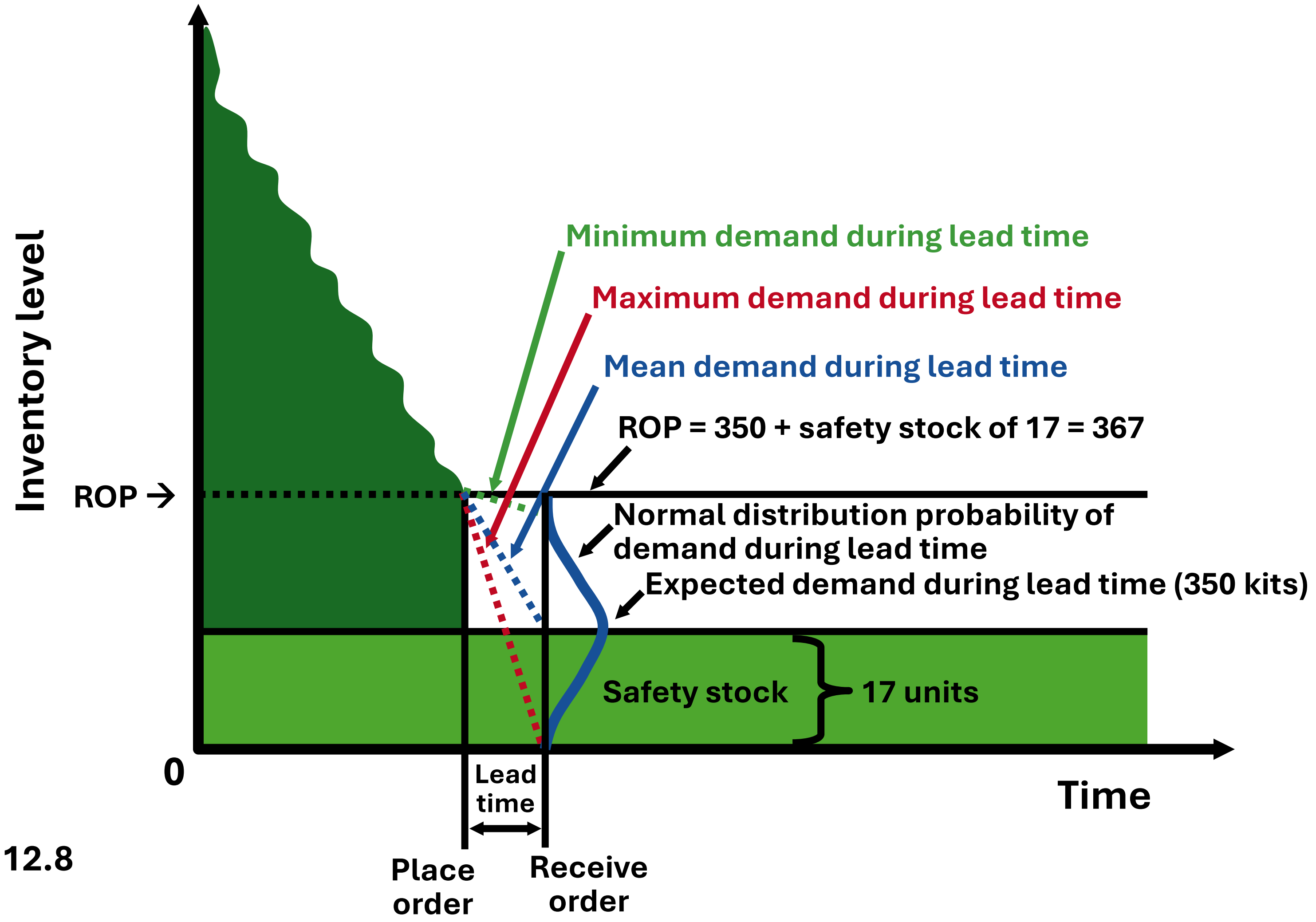

Figure 12.8

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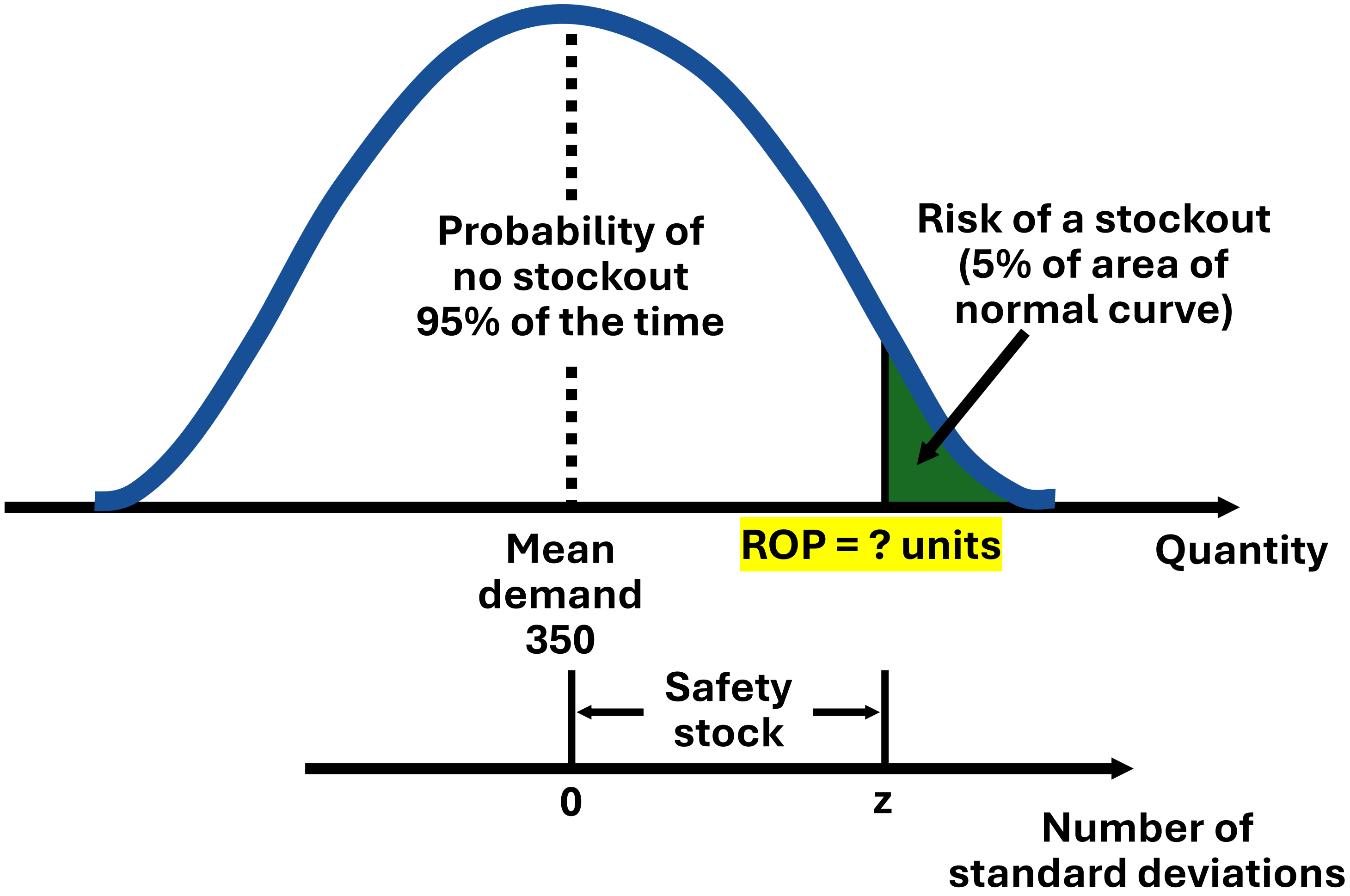
Probabilistic Demand



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Example

Average demand during lead time = μ = 350 units
Standard deviation of demand during lead time = σ_{dLT} = 10 units
5% stockout policy (therefore Service Level = 95%):

- Using the Normal Distribution Table, for an area under the curve of 95%, the ***Z*=1.65**

Always roundup

- Safety stock = $Z\sigma_{dLT}$ = 1.65(10) = 16.5 = 17 units
- **Reorder point** = Expected demand during lead time + Safety stock

  = 350 units + 17 units of safety stock

  = **367 units**

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Other Probabilistic Models

When data on Demand during Lead Time is not available, there are other models available

1. **Demand is variable and Lead Time is constant**
2. **Lead Time is variable and Demand is constant**
3. **Both Demand and Lead Time are variable**

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Other Probabilistic Models

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Other Probabilistic Models

1. Demand is variable and Lead Time is constant

Safety Stock

ROP = (average daily demand x lead time) + $Z \times \sigma_{dLT}$

where, σ_d = stantard deviation of demand

$$\sigma_{dLT} = \sqrt{LT \times \sigma_d^2} = \sqrt{LT} \times \sigma_d$$

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Other Probabilistic Models

Example: Probabilistic Demand and Lead Time constant

Average daily demand (normally distributed) = 15 units;
Standard deviation = 5 units;
Lead time is constant at 2 days;
90% service level desired.

Reorder Point (ROP)?

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Other Probabilistic Models

Example: Probabilistic Demand and Lead Time constant

Average daily demand (normally distributed) = 15 units
Standard deviation = 5 units
Lead time is constant at 2 days
90% service level desired

***Z* for 90% = 1.29 from ND Table**

ROP $= (15 \text{ units} \times 2 \text{ days}) + Z\,\sigma_{dLT}$

$= [30 + Z \times (\sqrt{LT} \times \sigma_d)] = 30 + 1.29(\sqrt{2})(5)$

$= 30 + 9.12 = 39.12 \approx 40$ units

ROP = 40 units, and Safety Stock is 10 units

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Other Probabilistic Models

2. Lead Time is variable and Demand is constant

ROP = (daily demand x average lead time) + $Z \times \sigma_{dLT}$

where, σ_{LT} = standard deviation of lead time in days

$$\boldsymbol{\sigma_{dLT} = \sqrt{d^2 \times \sigma_{LT}^2} = d \times \sigma_{LT}}$$

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Other Probabilistic Models

Example: Probabilistic Lead Time and Demand constant

Daily demand (constant) = 10 units;
Average lead time = 6 days;
Standard deviation of lead time = σ_{LT} = 3 days;
Service level desired = 98%.

Reorder Point (ROP)?

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Other Probabilistic Models

Example: Probabilistic Lead Time and Demand constant

Daily demand (constant) = 10 units
Average lead time = 6 days
Standard deviation of lead time = σ_{LT} = 3 days
Service level desired = 98%

***Z* for 98% = 2.06 From ND Table**

ROP = (10 units x 6 days) + (Z x $d \times \sigma_{LT}$)
= 60 + 2.06(10)(3)
= 60 + 61.8 = 121.8 = 122 units

ROP = 122 units, and Safety Stock is 62 units

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Other Probabilistic Models

3. Both Demand and Lead Time are variable

ROP = (average daily demand x average lead time) + $Z \times \sigma_{dLT}$

where σ_d = standard deviation of demand per day

σ_{LT} = standard deviation of lead time in days

$$\sigma_{dLT} = \sqrt{\mu_d^2 \times \sigma_{LT}^2 + \mu_{LT} \times \sigma_d^2}$$

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Other Probabilistic Models

Example: Probabilistic Lead Time and probabilistic Demand

Average daily demand (normally distributed) = 150 units;
Standard deviation = σ_d = 16 units;
Average lead time 5 days (normally distributed);
Standard deviation = σ_{LT} = 1 day;
95% service level desired.

Reorder Point (ROP)?

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Other Probabilistic Models

Example: Probabilistic Lead Time and probabilistic Demand

Average daily demand (normally distributed) = 150 units
Standard deviation = σ_d = 16 units
Average lead time 5 days (normally distributed)
Standard deviation = σ_{LT} = 1 day
95% service level desired

Z for 95% = 1.65 from ND Table

ROP = (150 units x 5 days) + Z x σ_{dLT} *with* $\sigma_{dLT} = \sqrt{\mu_d^2 \times \sigma_{LT}^2 + \mu_{LT} \times \sigma_d^2}$

$= (150 \times 5) + 1{,}65\sqrt{(5 \times 16^2) + (150^2 \times 1^2)}$

= 750 + 1.65(154.2) = 1004.44 ➔ **1005 units**

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